

University of Bahrain

College of Information Technology

Department of Computer Science

ITCS252 Discrete Structure I

Second Semester 2013/2014

Exam #2 — One Hour

NAME	[REDACTED]
ID #	[REDACTED]
SECTION #	[REDACTED]
SERIAL	[REDACTED]

Time: 2:00 – 3:00 PM

This exam contains 4 pages (including this cover page) and 6 questions. Check to see if any pages are missing. Enter all requested information.

You are not allowed to use books, notes, or mobiles

Question	Points	Score
1	8	6
2	6	6
3	6	6
4	8	7
5	6	6
6	6	6
Total:	40	37

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Question	Points	Score
1	8	4
2	8	5.5
3	8	6.75
4	10	10
Total:	34	26.25

(1) [8 points] Answer the following questions.

(a) Give a counter example to: $\forall m, n$: if both m and n are distinct primes, then $m + n$ is prime.

2 $m = 5, n = 3$
 $m + n = 5 + 3 = 8 \Rightarrow 8$ is not prime

(b) Give a counter example to: if n is perfect square, then $n + 4$ is perfect square.

2 $n = 9 = (3)^2$
 $n + 4 = 9 + 4 = 13 \Rightarrow 13$ is not a perfect square

(c) The product of two even integers is composite. Write the

2 Hypothesis = n, m are even integers
 Conclusion = The product of n and m ($n \cdot m$) is composite

(d) True or False, Explain. $\forall n \in \mathbb{Z} \sqrt{-3n^2 - n(6 - 4n) + 9}$ is rational

False \times counter example let $n = 1$
 $\sqrt{-3(1)^2 - 1(6 - 4(1)) + 9} = \sqrt{2}$
 $\sqrt{2}$ is irrational
 $n = 1 \Rightarrow -3 - (6 - 2) + 9 = -5 + 9 = 4$

(2) [6 points] Show that the difference of the squares of two consecutive integers is odd.

6 Direct proof: Assume there is two consecutive integers n and m .
 any definition $n = k, m = k + 1, k \in \mathbb{Z}$
 any substitution
 $m^2 - n^2 = (k + 1)^2 - k^2 = k^2 + 2k + 1 - k^2$
 $= 2k + 1, k \in \mathbb{Z}$
 \therefore odd $\#$

- (3) [6 points] Show by contradiction that for any integer n : if $n > 0$, then $\frac{n}{n+1} > \frac{n}{n+2}$

Suppose not. Suppose there exists an integer n such that
 $n > 0$ and $\frac{n}{n+1} \leq \frac{n}{n+2}$

$$n(n+2) \leq n(n+1)$$

$$n^2 + 2n \leq n^2 + n$$

$$n^2 - n^2 + 2n - n \leq 0$$

$$n \leq 0 \quad (\text{contradiction with } n > 0)$$

- (4) [8 points] Let $U = \{x \in \mathbb{Z} \mid -5 \leq x \leq 5\}$, $A = \{1, 2\}$, $B = \{1, 2, 5\}$ and $C = \{1, 3, 5\}$. Answer the following questions. Show your work.

- (a) Find $(A \cap C) \cup B$

$$= \{1, 2, 3, 5\} \cup \{1, 2, 5, 5\}$$

$$= \{1, 2, 3, 5\} \cup \{1, 2, 5\} = \{1, 2, 3, 5\}$$

$$= \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

- (b) Find $P(A) \cap P(B - C)$

$$P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

$$P(B - C) = \{\emptyset, \{2\}, \{5\}\}$$

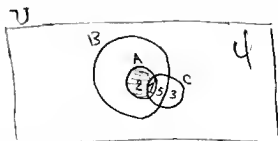
$$P(A) \cap P(B - C) = \{\emptyset, \{2\}, \{5\}\}$$

- (c) Find $|P(A - B) \cup P(B - A)|$

$$= |P(\{5\}) \cup P(\{1, 2\})| = |P(\{5\})| + |P(\{1, 2\})| - |P(\{5\}) \cap P(\{1, 2\})|$$

How

- (d) Draw Venn Diagram for $(A \cap B) - C$. Show all numbers of the U set in the Diagram.



- (5) [6 points] show that $(C - D) \cup (C \cap D) \subseteq C$

$$\text{let } x \in (C - D) \cup (C \cap D)$$

$$\therefore x \in (C - D) \text{ or } x \in (C \cap D)$$

$$\text{Case 1: } x \in (C - D)$$

$$x \in (C \cap \bar{D})$$

$$\therefore x \in C \text{ and } x \notin D \Rightarrow x \in C$$

$$\text{Case 2: } x \in (C \cap D)$$

$$x \in C \text{ and } x \in D$$

$$\text{then } x \text{ is rejected because } x \in D \Rightarrow x \in C$$

- (6) [6 points] Show that $(A - B) \cup (A - C) = A - (B \cap C)$

$$\text{R.H.S.} = A - (B \cap C)$$

$$= A \cap (\overline{B \cap C})$$

$$= A \cap (\bar{B} \cup \bar{C}) = (A \cap \bar{B}) \cup (A \cap \bar{C})$$

$$= (A - B) \cup (A - C) = \text{L.H.S.}$$